Nurturing Markets, Widening Horizons

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Note on Cubic Spline Valuation Methodology

THE CUBIC SPLINE METHODOLOGY

A model for yield curve takes traded yields for available tenors as input and generates the curve through interpolation and curve fitting, so as to minimize the error between traded and model prices. Cubic Spline methodology has been chosen by FIMMDA as it allows minimum error while giving a smooth, continuous curve, which is essential for correct pricing of debt securities. The technical details of the yield curve construction, optimization, smoothing is given in Annexure 1.

A. Process of the methodology updated **till November 2014**

The methodology for generation of the yield curve and the valuation for G-Secs are Highlighted below: -

a) **Identification of Nodal Points*****: On the first working day of every month, the FIMMDA Valuation Committee would identify "**Nodal Points**" (one bond per calendar year tenor) from the outstanding stock of Government of India Securities. A GOI Security would qualify to be a "**Nodal Points**" if it qualifies the following criteria:

i. There will be only ONE **Nodal Point** for a calendar year of maturity (2012, 2013, 2014 …………2041, etc.).

ii. The "**Nodal Points**" should have had a minimum number of 100 trades and minimum volume of Rs.1000 Crores traded both on the NDS-OM and PDO-NDS, in the immediate preceding month.

iii. If a **Nodal Point** security fails to meet the above criteria in the subsequent month, it would still qualify to be a **Nodal Point** if it meets the criteria of 50 trades and 500 Cr, volume. (for the subsequent month only)

b) **Identification of "Nodal Point** or Nodal Point*"**: As there are certain "important input tenors" for getting a realistic Yield Curve (viz. 1 to 7 year, and 10 years), it would be necessary to identify GOI securities called "**Nodal Point** or Nodal Point***" in the tenors 1 to 7 years and 10 years in case they do not satisfy the criteria to be a "**Nodal Point*****". A "**Nodal Point** or Nodal Point***" is therefore a security in a particular calendar year tenor (1 -7 years and 10 years) which does not have the minimum trades and volumes (100 and Rs1000 crores) to qualify for a "**Nodal Point*****", for the aforementioned "important input tenor" The **Nodal Point** or Nodal Point*** for the aforementioned tenors would be one with the maximum number and amount of trades in that tenor. These inputs are required, so that the Model Generated Yield Curve is more representative of market behavior.

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c) **Selection of Bonds for curve construction**: From the Universe of all outstanding bonds :

i. For each year only one bond would be taken for curve construction.

ii. The bond selected for curve construction should have traded during the day, and should preferably be a **Nodal Point*** or Nodal Point** or Nodal Point*** as mentioned in step (a) and (b) above.

iii. A traded bond should have traded a minimum number of trades and minimum volume of trades as decided by the "Filter" set for the month by the Valuation Committee.

Filter: For substituting the model prices with the LTPs for a **Nodal Point** and non-**Nodal Point** security, a daily criterion is calculated.
Identify the **Nodal Point** with largest number of Trades (T1) and Volumes (V1) and also the **Nodal Point** with Minimum Number of Trades (T2) and Volumes (V2).
The daily criterion is a ratio of:
% of T2/T1
% of V2/V1

Modification in the computation of Ratios for criteria based selection of LTP/LTY: *A 3 month moving average of the ratio calculation as above will be used for the computation criteria for LTP/LTY.*

However, there is no filter criteria for **Nodal Point**, *but if a nodal point is not crossing the day-end filter criteria, and if it is traded after 3 pm, it would be recognized as an input in the model for curve generation and valuation. If there is any trade before 3 pm and not crossing the filter criteria, it will not be used for 'input' but after the yield curve is generated, the value obtained for the nodal point security as per yield curve, will be replaced by the actual price at which the security was traded before 3 pm.*
Note: Considering the time stipulation for reporting the OTC deals on Reported Deals section, they will be considered for valuation, in the absence of sufficient trades on NDS-OM.

iv. For tenors 1-7 years and 10 years ("important input tenors"), if the "**Nodal Point**" identified, has not traded on a particular day, but some other bond in these tenors has traded and passed the "Filter" criteria, then that bond's yield and price would be used for curve construction.

v. With a view to adopt the FASB and IFRS Level 1, 2, and 3 criteria, the level 2 criteria of Fair Valuation is adopted. Thus in case there are no trades in any **Nodal**

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Point on a particular day, a "**Market Observable & Tradable**" MOT input would be used. The "Market Observable and Tradable" input is one in which the **Nodal Point** has "bids" and "offers" aggregating a minimum of Rs.15 crores each and a maximum spread of 10 basis points at 12 noon, 2pm and 4 pm. The weighted average "mid-yield" of the particular security across the three time-frames would then be used as an input for curve construction.

For IIB the minimum bid/offer can be Rs.5 crores each at only two times: 12 noon and 4 pm.

(And for FRB please refer to the document Valuation of FRBs under G Sec Valuation tab on FIMMDA's website: www.fimmda.org)

vi. For tenors 1-7 years and 10 years ("important input tenors"), and for other tenors (for which "**Nodal Point**" bonds had been identified on the first working day of the month) if no **Nodal Point** has traded, or no other traded bond which has passed the "filter" criteria is available, or a Market Observable & Tradable input is not available, "**Proxy Yields**" would have to be used for the selected **Nodal Point** securities.(Computation of Proxy Yields is dealt with in subsequent paragraphs).

vii. For tenors 1-7 years and 10 years ("important input tenors"), Market Observable & Tradable" (MOT) inputs if available, would be used for "input" provided:

a) The MOT passes the traded data "Filter" criteria (total of bid and offer amount to be equal or greater than the Filter amount, and total number of bids and offers to be equal or higher than the Filter number of trades), and the MOT is available with 10 bp spreads in the specified times.

b) The total of traded numbers and amount PLUS the MOT passes the traded "Filter Criteria".

viii. For other tenors i.e. 8 yrs., 9 yrs., and above 10 yrs., any bond (whether **Nodal Point** or not) which has traded and passed the "Filter" criteria would be used as an input for Curve Construction (provided in the particular tenor there is no other security which has sufficient trades to qualify for inputting during curve construction). If more than one bond has traded in a particular tenor, the bond with the highest number of trades on the particular day would be taken.

ix. For other tenors i.e. 8 yrs., 9 yrs., and above 10 yrs., any bond (whether **Nodal Point** or not), "Market Observable & Tradable" (MOT) inputs if available, would be used for "input" provided:

a) The MOT passes the traded data "Filter" criteria (total of bid and offer amount to be equal or greater than the Filter amount, and total number of bids and offers to be equal or higher than the Filter number of trades), and the MOT is available with 10 bp spreads in the specified times.

b) The total of traded numbers and amount PLUS the MOT passes the traded "Filter Criteria.

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x. Adjustment for 30-year tenor point: The curve needs to be generated till 30 years. If the 30-year **Nodal Point** Bond does not trade on a particular day, then the difference between its last traded yields (provided it was traded in the past 14 days) and the model generated 20-year par-yield for the corresponding day is calculated. This difference is added to the current day's 20-year model generated par-yield to arrive at the 30-year **Nodal Point** Bond yield. If the bond did not trade in the last 14 working days then the yield of the farthest tenor traded bond of the current day would be taken as the yield of the 30-year **Nodal Point** Bond.

xi. Other criteria: The other criteria in selecting inputs for the curve construction are

- If a new bond has been issued during the month and the bond meets FIMMDA's criteria for being a **Nodal Point** Bond (but not designated as such) then the new bond would be taken into curve construction, if:
  a) It passes the "Day-End Filter Criteria".
  b) There is no other paper in the respective tenor exist or has not been used as an "Input Point" (i.e. either a **Nodal Point*****/ **Nodal Point** or Nodal Point***)
  c) If an existing paper qualifying for a **"Nodal Point"** is already trading, then
     i. The yield of the new paper is "Lower" than the existing **Nodal Point**
     ii. The yield of the new paper is "Higher" than the existing **Nodal Point**, but the volume and number of trades are more than existing security.

  In all other cases, the existing security will continue to be an input point.

- For the tenors less than1-year tenor, the lowest tenor traded T-Bill would be used and it would be extrapolated till overnight period
- Only G-Secs without features like floating coupon, embedded options etc. would be used as inputs for curve construction.

B. Base liquid zero rate curve and par-yield curve generation: - Based on the above data, a base liquid zero rate curve based on Cubic Spline approach is generated. Further a smoothing technique is applied to ensure that the forward rate curve is smooth. The par-yield curve is generated from the zero-rate curve.

C. Computation of illiquidity: - The illiquidity factor is calculated based on the yield differential between the yield of a traded bond and the model generated par yield for the same bond's residual tenor on that day. It would be generated every Tuesday on a 4-week moving average basis. The process of calculating the illiquidity factor is elaborated in "Illiquidity Factor" below.

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D. Valuation & Substitution of Model Prices

"Market Observable & Tradable" (MOT) inputs if available, would be used for substituting model prices for bonds which have not traded. The "Market Observable and Tradable " input is one in which the bond has "bids" and "offers" aggregating a minimum of Rs.15 crores and a maximum spread of 10 basis points at 12 noon, 2 pm and 4 pm. The weighted average "mid-yield" of the particular security across the three time-frames would then be used to substitute the model price. (This is done to give a more market related shape to the yield curve, in tenor segments where lack of traded inputs leads to the Cubic Spline model tending to give an unrealistic curvature "trough" or " hump")

The conditions for using MOT as for "substitution" would be as under:

i. The MOT would need to be available at three time periods 12 noon, 2 pm, and 4 pm (subject to change).

ii. For **Nodal Points**, maximum spread of 10 bps and Rs.15 crores (bid–offer total) would be required.

iii. For all other bonds, the total number of bids and offers and total of bid and offer amount with spread of 10 bps, and total amount equal to or above the minimum "Filter" set for recognition of traded prices as input and substitution, would be required.

iv. **For other tenors** i.e. 8 yrs., 9 yrs., and above 10 yrs., any bond (whether **Nodal Point*** or Nodal Point** or Nodal Point*** or not), "Market Observable & Tradable" (MOT) data if available, would be used for "substitution" provided:
   a. The MOT passes the traded data "Filter" criteria (total of bid and offer amount to be equal or greater than the Filter amount, and total number of bids and offers to be equal or higher than the Filter number of trades), and the MOT is available with 10 bp spreads in the specified times.
   b. The total of traded numbers and amount PLUS the MOT passes the traded "Filter Criteria.

v. The procedure outlined in (d) above would be used even for those bonds which are not **Nodal Points**, but are in tenors in which these **Nodal Points** exist. (E.g. 8.08 % 2022, and 8.13 % - 2022 in the 2022 tenor, where the **Nodal Points** is 8.15 % -2022; or 7.80 %- 2021, and 10.25 % - 2021 in the 2021 tenor, where the **Nodal Point** is 8.89 % -2021).

Proxy yield

For each year between tenors 1 to 7 years and 10 years, a yield must be taken for base curve calibration. This is needed as the steepness in the curve between each of these tenors changes significantly. If for any year (in the tenors 1 to 7 and 10 years) the **Nodal Point** does

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not trade on a particular day then proxy yield for that tenor has to be generated. Proxy yield would be generated as follows:

For **Nodal Point** that did not get traded (in the tenors 1 to 7 and 10 years), the proxy yield would be calculated by adding a factor to that bond's traded/proxy yield of the previous day. The factor would be calculated as follows:

a. Difference in yield is computed for the traded **Nodal Point** security of the tenor immediately preceding the tenor for which proxy yield is required. Similar difference in yield is computed for immediately succeeding tenor
b. Average of the difference in yield of the of the two tenors (traded on the day) is computed as the factor
c. If no preceding **Nodal Point** is traded (T-Bill is not considered for this calculation), then the factor would be the difference in yield of the immediate succeeding traded **Nodal Point**; or
d. If no succeeding **Nodal Point** is traded (T-Bill is not considered for this calculation), then the factor would be the difference in yield of the immediate preceding traded **Nodal Point**.

For calculating proxy yields, only tenors of 1-7, and 10 years would be considered.

Illiquidity factor

The illiquidity factor would be calculated as below:

- For each bond the illiquidity value is calculated as the difference in the traded yield and model generated liquid par-yield for the bond's residual tenor. Sample calculation for the G-Secs maturing in 2012 as on June 11, 2010 is shown below:

Bond	Maturity Date	Actual Traded Yield	Interpolated Par Yield from model curve	Illiquidity value (in bps)
6.85	04/05/12	N.T		
7.40	05/03/12	6.05%	6.05%	0.00
10.25	06/01/12	N.T		
11.03	07/18/12	6.21%	6.12%	9.00
9.40	09/11/12	6.23%	6.17%	6.00
4.63	11/10/2012	N.T		

- Similar exercise is done daily and the moving average for the past 4 weeks is calculated for each security. Further, an average of positive illiquidity spreads for all the bonds maturing in a particular tenor is also calculated. For example, in the illustration above, an average of the illiquidity spreads would be taken for all bonds maturing in 2012. The averages calculated are floored to zero. This is done to ensure that no bond has a negative illiquidity factor.
- If a particular bond has traded for more than 5 days in the past 4 weeks then the 4-week average illiquidity value calculated for that particular bond would be used as the illiquidity factor.
- If a particular bond has traded for less than 5 days in the past 4-weeks, then the average illiquidity value calculated for all the bonds maturing in a particular tenor would be used as the illiquidity factor.

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- 4 week moving averages would be calculated on every Tuesday based on the immediate preceding 4 weeks and applied for the valuations during the week.
- If none of the bonds maturing in a particular tenor have traded in the past 4 weeks then the average of the illiquidity factor of immediate next and immediate previous tenors would be used. For example if none of the bonds maturing in 2017 have traded in the last 4 weeks then illiquidity factor would be calculated as the average of illiquidity factor for 2016 (say 8 bps) and 2018 (say 2 bps) i.e. 5 bps. When there are no trades beyond a particular tenor say beyond 2027, then the illiquidity factor applicable to the immediate preceding available traded tenor, say, 2027 would be applied.

Special Dispensations regarding Illiquidity Factor:

When a new paper is issued, *or when there are more than one paper traded in a particular tenor, one qualifying for a Nodal Point input, and the other passing the "Filter Criteria" for recognition of the traded price for valuation purposes,* the illiquidity factor would be calculated as follows:

a. The "Par-Yields" will be generated after inputting the new/*Nodal Point Qualifier* in the CS model.
b. The difference between the traded /reported prices (on NDS-OM) in the same tenor papers and the respective "Par-Yields" would be the "Illiquidity Factor". In case *two or more* papers are traded in the same tenor the IFs will be averaged ignoring the lowest IF amongst the traded securities in the particular tenor.
c. The issue of a new paper/*more than one paper trading in a given tenor*, results in widening the "Illiquidity Factor "for *outstanding* papers of similar tenor. Therefore, a 3 –Day Moving Average of the IFs calculated as above, would be applied to the existing papers instead of the normal 4-Week Moving Average .
d. If, after adding the "Illiquidity Factor", the non-traded bonds in a particular tenor show a yield which is lower than the yield of a traded bond whose volume and trades have passed the filter for recognition for valuation purposes, the model – determined yield (including the IF) would be increased to equal the yield of the traded bond. Thus:
   Yield of a non-traded bond in a particular tenor would be = or > than the yield of the traded bond with the lowest yield in that tenor.
e. If there are no trades in a particular tenor, the Model Yield plus the IF would give the final yield for valuation of bonds in that tenor.
f. If the outstanding papers in a tenor become totally illiquid (i.e. no trades in a day) the last IF would continue to be used till the immediately next quarter end.

To Summarise

1. The methodology of curve construction is broadly two steps
   Step 1: Select "Input" Tenors
   Step 2: (i) Substitute traded and Minimum Observable Tradable (MOT) prices in Model Generated Curve
   (ii) Add Illiquidity factors for non-input bonds.

STEP 1

2. "Mandatory" input tenors are: 91 DTB, 1-7 yrs and 10 yrs.
3. Additional inputs
    a. "**Nodal Point**" bonds selected at the beginning of the month. (which may or may not be in 1-7 yrs and 10 yrs)
    b. Traded bonds which pass a minimum criteria (Filter)
    c. Minimum Observable Tradable data (MOT)
    d. MOT plus sparsely traded data total of which crosses minimum criteria (Filter)
4. If no inputs are available for Step 1, calculate the "Proxy Yields".

STEP 2

5. All "Input" prices/yields are substituted in the Model generated curve (including the ones from 'point 3')
6. Calculate "Illiquidity Factors" – 4 week Moving Average or 3 days Moving Average and add to the Model Price of all "non – input "securities for arriving at the closing valuation price and yield.

FRB Valuation: Please refer the separate item under the G SEC VALUATIONS

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Annexure 1

A detailed description of the Pienaar Choudhry method for extraction Zero rates and Par Yields from Traded Bond Prices

FIMMDA was entrusted with the task of developing a suitable model for the yield curve generation and streamlining the process for arriving at the prices for the G-Secs. Nelson Siegel Svensson and cubic spline zero curve were considered. A model based on Nelson Siegel Svensson provides a smooth zero curve; however it suffers from the demerit of a relatively higher price errors. This is because the model cannot incorporate multiple changes in curvature across various tenors. A cubic spline curve was considered to be appropriate for the Indian markets as the curve tracks the input price of various tenors and thereby produces a lower model error. In this approach the traded or proxy yields serve as the input for curve construction and a cubic spline is used to interpolate between the input yields to generate the curve. The cubic spline is a series of curves that is continuous at all the points. Each curve of the spline is of third order and has the form $Y = ax^3 + bx^2 + cx + d$ *where Y is zero-rate for the tenor 'x'.*

In the current method an optimization function is used to fit a natural cubic spline based zero curve to a set of traded bond prices. A yield curve is generated from the cubic spline based zero curve. The base research paper used as reference is authored by Rod Peinaar and Moorad Choudhry[1]. A simple cubic spline based implementation gives a good fit but leads to wavy forward curve. Hence a further smoothening constraint is applied to the optimization procedure that generates a curve which has minimum curvature and minimum price error. This smoothening leads to a better behavior of forward rates extracted from the zero rate curve.

The zero rate curve thus obtained is also used to extract par-yields for different maturities.

What is a cubic spline function?

A cubic spline function is a piecewise cubic polynomial function that passes through a given set of points in a smooth fashion. . The function takes the form $f = a_i + b_i\Delta + c_i\Delta^2 + d_i\Delta^3$; where 'i' represents the portion of the time axis where we want to measure zero-rate. If T_i represents the time to maturity of a traded bond, then between T_i and T_{i+1}, Δ takes values from 0 to $T_{i+1} - T_i$.

The time axis is divided in to regions by "knot points" at times T_i (usually the traded bond maturity in years). As we can see there is a different set of coefficients (a_i, b_i, c_i, d_i ...) describing the zero-rate curve between every T_i and T_{i+1}. The value of the cubic spline function as well as its first and second derivatives are the same when measured from either side of the knot point.

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For example, consider three consecutively maturing bonds, with maturity dates 14/06/2015, 17/08/2016 and 28/08/2017.

The current date is 29/07/2010; the time to maturity for each of these bonds is 4.875 years, 6.05 and 7.0805556 years respectively. So within the cubic spline framework the zero-rate between 4.875 and 6.05 years is described by one set of coefficients (a_i, b_i, c_i, d_i ...) and between 6.05 years and 7.08 years is described by another set of parameters ($a_{i+1}, b_{i+1}, c_{i+1}, d_{i+1}$ ...). The value of Δ varies from 0 to (6.05-4.875=1.175) between 4.875 and 6.05 years. Similarly Δ takes the values from 0 to 1.0305 between 6.05 and 7.08 years. The zero-rate for 4.875 years is a_i, while that for 6.05 years is a_{i+1}.

Based on the constraints applicable for natural cubic spline (discussed in Annexure 1(a)) it is possible to describe the (b_i, c_i, d_i ...) coefficients in terms of the a_i coefficients and the maturities T_i. So in the optimization set-up the problem is to find values of a_i such that the squared difference between the model generated prices and traded prices is least

Pricing a coupon-bond given a zero rate curve

Suppose the values of the a_i coefficients for the cubic spline are as follows

Dates	a_i coefficients
29/07/2010	5.60%
02/07/2011	6.108%
03/09/2013	7.047%
14/06/2015	7.550%
17/08/2016	7.712%
28/08/2017	7.762%
03/05/2020	7.776%
15/02/2022	8.064%
02/07/2040	8.331%

Then as discussed earlier it is possible to obtain zero-rates (guesses) for any maturity. Consider the bond with coupon 9.39% and maturity date 02/07/2011. The (model) price for this bond is obtained using discount factors obtained from zero rates (see diagram below).

9.39 bond maturity i.e. 02/07/2011

Discount factors obtained by converting zero-rates

Coupon Dates	Coupons	DF (discount)	PV for coupon
02/01/2011	4.695	0.975503799	4.57999033
02/07/2011	104.695	0.945663949	99.0062871

Dirty Price	103.586277
Accrued interest	0.70425
Clean Price	102.882027

This clean price is the model price of the coupon bond.

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Pricing a T-bill given a zero-rate curve

The price of the T-bill is taken as 100 times the discount factor (from zero rates) for the maturity of the T-bill.

Fitting the zero-curve to reproduce traded bond prices

Let P represent the traded bond price (either T-bill or coupon bondand $\hat{P}$ model price (produced as above using a guess of the zero-curve). The values of coeficients (b_i, c_i, d_i ...) can be found from coefficent a_i. So by modifying a_i coefficient (using an optimization algorithmLevenberg Marquardt algorithm has been used in the current implementation) the cumulative price difference_$\sum(\hat{P}-P)^2$_between the model price and traded price of the traded bonds is minimized.

Smoothening of zero-rate curve

To ensure that smooth forward rates are obtained from the zero-rate curve, a curvature term is added to the minimization of price differences. So instead of minimizing $\sum(\hat{P}-P)^2$ (bond price difference), $\sum(\hat{P}-P)^2 + \int \lambda(t) * [f'']^2$ __ is minimized. The start point for this minimization is taken as the curve obtained by simply matching bond prices from the previous step.

f represents the zero-rate curve and f'' the second derivative of the curve.

Here **λ (t)** ('t' being time in years or maturity of the traded bond) is a function that augments the curvature, it is also called the VRP (variable roughness penalty) function.

Using this form of minimization leads to smoother curve but with possible mismatch in model price of bonds and traded price of bonds.

In the current implementation **λ(t)** is same as suggested by Daniel F. Waggoner[2]. It is a stepwise function and takes the following values:

$$\lambda(t) = 0.1\_for\_t < 1$$
$$\lambda(t) = 100\_for\_1 \le t < 10$$
$$\lambda(t) = 100000\_for\_10 \le t$$

*Results:*Once the zero rate is obtained by the above method the par yield is derived from it. Par yield (or par rate) is the coupon rate for which the price of a coupon bond is equal to its par-value. For various maturities such as 0.25 years, 0.5 years..., let C represent the par-yield. Then one can solve the equation $C = \frac{1 - df_{mat}}{\sum_k df_k}$, to obtain the par-yield for that maturity. Here df_{mat} represents the discount factor for the maturity date of the imaginary bond (say for 1.5 years maturity) and df_k represents the discount factors for the K-th coupon payment date of this imaginary coupon-bond. Again the discount factors are obtained from the zero-rate curve obtained from optimization earlier.

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Appendix 1(a)

Obtaining other coefficients from a_i coefficients

Matching the values of zero-rate, the first derivative of zero rate and the second derivative of the zero rate at the knot points T_i it is possible to write the following equations:

$$a_{i+1} = a_i + b_i\Delta + c_i\Delta^2 + d_i\Delta^3$$

$$b_{i+1} = b_i + \Delta_i(c_{i+1} - c_i) \quad \text{- (A)}$$

$$d_i = \frac{c_{i+1} - c_i}{3\Delta_i} \quad \text{- (B)}$$

For the coefficient c_i the following recurrence relation can be written:

$$\Delta_{i+1}c_{i+2} + \Delta_i c_i + 2(\Delta_i + \Delta_{i+1})c_{i+1} = -3\{(\frac{a_{i+1} - a_i}{\Delta_i}) - (\frac{a_{i+2} - a_{i+1}}{\Delta_{i+1}})\} \quad \text{- (C)}$$

Furthermore if there are in all "N" bonds, the values of c_1 and c_N are taken as zero (these conditions are called Natural cubic spline conditions), then the recurrence relation for the c_i coefficient s can be rewritten in the matrix form as a set of linear equations:

$$[2(\Delta_1 + \Delta_2)\ \ \Delta_2 \ldots\ldots\ ] = [-3\{(\frac{a_2 - a_1}{\Delta_1}) - (\frac{a_3 - a_2}{\Delta_2})\}]$$

$$[\ \ \Delta_2\ \ 2(\Delta_2 + \Delta_2)\ \Delta_3 \ldots\ ] = [-3\{(\frac{a_3 - a_2}{\Delta_2}) - (\frac{a_4 - a_3}{\Delta_3})\}]$$

$$[\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{- (D)}$$

$$[\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ] = [\ldots\ldots\ldots]$$

$$[\ldots\ldots\ldots\ldots\ldots\ldots\Delta_{N-2}\ 2(\Delta_{N-2} + \Delta_{N-1})] = [-3\{(\frac{a_{N-1} - a_{N-2}}{\Delta_{N-2}}) - (\frac{a_N - a_{N-1}}{\Delta_{N-1}})\}]$$

From the set of linear equations **(D)** one can find $\begin{bmatrix} c_2 \\ c_3 \\ . \\ . \\ . \\ . \\ c_{N-1} \end{bmatrix}$ using matrix mathetics (see reference to tridiagonal system of matirices).

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Thus as a first step the c_i coefficient s are obtained from the a_i coefficient s and then the remaining coefficients are obtaied using equations (A) and (B).
Once all coefficients are obtained we have our zero-rate curve.

References:

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